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From September 21, 1886, to April 1, 1888, is $1\frac{19}{36}$ years.
From September 21, 1886, to April 1, 1889, is $2\frac{19}{36}$ years.
From September 21, 1886, to April 1, 1890, is $3\frac{19}{36}$ years.
Let x=amount paid for first note ; y, for second ; z, for third.
$\therefore$ $x(1.08)^{1\frac{19}{36}}=1240$, or $\log x=\log 1240-1\frac{19}{36}\log 1.08$.
$\therefore$ $x=\$1102.448$.
$y(1.08)^{2\frac{19}{36}}=1300$, or $\log y=\log 1300-2\frac{19}{36}\log 1.08$.
$\therefore$ $y=\$1070.176$.
$\log z=\log 621.52-3\frac{19}{36}\log 1.08$.
$\therefore$ $z=\$473.743$.
$x+y+z=\$2646.367$=whole amount to be paid for the notes.
(II). If the notes bear compound interest we get,
$\$1000\times(1.06)^4=\$1262.477$, amount of first note.
$\$1000\times(1.06)^5=\$1338.226$, amount of second note.
$\$457\times(1.06)^6=\$648.263$, amount of third note.
$\therefore$ $\log x=\log 1262.477-1\frac{19}{36}\log 1.08$.
$\therefore$ $x=\$1122.43$.
$\log y=\log 1338.226-2\frac{19}{36}\log 1.08$.
$\therefore$ $y=1101.646$.
$\log z=\log 648.263-3\frac{19}{36}\log 1.08$.
$\therefore$ $z=\$494.127$.
$x+y+z=\$2718.20$=whole amount paid for the three notes.

GEOMETRY.

Conducted by B. F. FINKEL, Springfield, Mo. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

78. **Proposed by J. A. MOORE, Ph. D., Professor of Mathematics, Millsaps College, Jackson, Miss.**

Required the number of normals that can be drawn from any point (a, b) to the parabola $y^2=2px$.

I. **Solution by the PROPOSER.**

The equation of the normal to the parabola in terms of its slope, (s), is

$$y=sx-\tfrac{1}{2}(sp)(2+s^2)\ldots\ldots\ldots\ldots\ldots\ldots(1).$$

Substituting a, b for x, y in (1), and putting the equation in a new form, we have,

$$s^3+\tfrac{1}{2}p(p-a)s+(2b/p)=0\ldots\ldots\ldots\ldots\ldots\ldots(2).$$

Denoting Sturm's functions by F, F_1, F_2, etc., we have the following:

$F=s^3+(2/p)(p-a)s+(2b/p)$.

$F_1=3s^2+(2/p)(p-a)$.

$F_2=-2(p-a)s-3b$.

$F_3=-b^2-(8/27p)(p-a)^3$.

Consider five cases.

(1). Suppose $p-a<0$, and $(8/27p)(p-a)^3$ numerically greater than b^2. Sturm's Theorem gives

	F	F_1	F_2	F_3
For $s=+\infty$,	+	+	+	+
$s=-\infty$,	−	+	−	+

Hence the roots are real and unequal.

(2). Suppose $p-a<0$, and $(8/27p)(p-a)^3$ numerically less than b^2. Then

	F	F_1	F_2	F_3
$s=+\infty$,	+	+	+	−
$s=-\infty$,	−	+	−	−

Hence, there is one real root.

(3). Suppose $p-a>0$. Then

	F	F_1	F_2	F_3
$s=+\infty$,	+	+	−	−
$s=-\infty$,	−	+	+	−

Hence, one real root.

(4). Suppose $-b^2-(8/27p)(p-a)^3=0$.

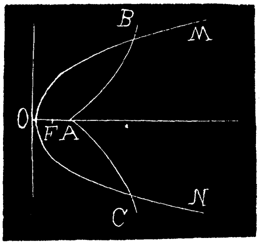
Then there are equal roots, as in this case F and F_1 have a common divisor, and all the roots are real.

(5). Suppose $-b^2-(8/27p)(p-a)^3=0$, and $p=a$.

Then $b=0$, and all the roots are equal, each being 0.

Hence if MON is the given parabola and BAC its evolute, that is, the semi cubical parabola whose equation is

$$b^2=\frac{8}{27p}(a-p)^3.$$



Then, (1), if the point (a, b) is within (to the right) of the evolute, three normals can be drawn to the parabola; (2), if the point (a, b) is on the evolute, but not at A, two normals can be drawn; (3), if the point (a, b) is A, or is without (to the left) of the evolute, one normal can be drawn to the parabola.

II. **Solution by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio, and OTTO CLAYTON, Fowler, Ind.**

If m be the tangent of the angle which the normal makes with the axis of x, the normal is given by

$$y=mx-pm-(\tfrac{1}{2}p)m^3 \qquad (1).$$

This passing through (a, b) gives

$$b=am-pm-(\tfrac{1}{2}p)m^3 \qquad (2),$$

a cubic in m, showing that the required number is three.

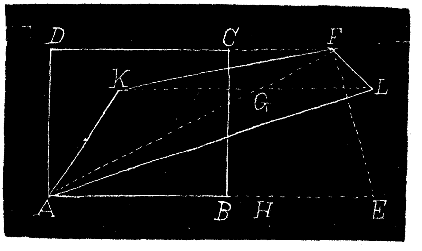
Also solved by *G. B. M. ZERR* and *J. F. LAWRENCE*.

79. **Proposed by JOHN MACNIE, Professor of Mathematics, University of North Dakota, University, N. D.**

To construct a quadrilateral of given area, the diagonals, one of which is given, cutting each other in given ratios and at a given angle.

I. **Solution by JAS. F. LAWRENCE, Freshman Class, Classical Course, Drury College, Springfield, Mo., and the PROPOSER.**

Let AC be a rectangle equivalent to the given area and having a side AB equal to one-half of the given diagonal. Produce AB to E, so that $BE=AB$; at A construct $\angle EAF$ equal to the angle to be made by the diagonals, and let AF meet DC produced in F. Divide AF in G in the ratio of division of one diagonal, and AE in H, in the ratio of the given diagonal. On an indefinite line drawn through G parallel to AB lay off GK, CL, equal to AH, HE, respectively, and FK, FL, AK, AL; $AKFL$ is the required quadrilateral.



Join EF. $\triangle FAE$ ⫏ AC. It is also equivalent to $AKFL$; for each is equivalent to one-half the parallelogram formed by drawing parallels through the extremities of the diagonals AF, LK.* Hence $AKFL$ ⫏ AC; it has also a diagonal $KL=AE=2AB$; its diagonals also are divided in the given ratios, and make an angle $FGL=\angle EAF=$the given angle. Hence, etc.

II. **Solution by G. B. M. ZERR, A. M., Ph. D., Lebanon, Va.; OTTO CLAYTON, Fowler, Ind.; and F. R. HONEY, Ph. B., New Haven, Conn.**

Let AB be the given diagonal, COB the given angle, $\triangle$ the given area, $m:n$ the given ratio for the known diagonal, $p:q$ the given ratio for the unknown diagonal.

***From the well known theorem: Any quadrangle is equivalent to one-half the parallelogram formed by drawing lines through its vertices parallel to its diagonals; follow the corollaries—**

a. **Two quadrangles are equivalent if their diagonals are respectively equal and intersect at the same angle. (Triangle a special case.)**

b. **Any quadrangle is equivalent to the rectangle of its diagonals multiplied by half the sine of their angle.**